NAME (underline family name):

STUDENT NUMBER: SIGNATURE:

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 263

ORDINARY DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

Examiner: G. Schmidt **Date:** Thurs. December 14, 2006
Associate Examiner: B. Charbonneau **Time:** 9:00 AM - 12:00 AM

Instructions

1. Write your name and student number on this examination script.
2. No books, calculators or notes allowed.
3. This examination booklet consists of this cover, 9 pages of questions and 2 blank pages (the cover page plus 11 numbered pages in all). Please take a couple of minutes in the beginning of the examination to scan the problems. (Please inform the invigilator if the booklet is defective.)
4. Answer all questions. You are expected to show all your work. All solutions are to be written on the page where the question is printed. You may continue your solutions on the facing page. If that space is exhausted you may continue on the blank pages at the end, clearly indicating any continuation on the page where the question is printed.
5. Your answers may contain expressions that cannot be computed without a calculator.
6. Use of a regular and or translation dictionary is permitted.
7. Circle your answers where confusion could arise.

GOOD LUCK!

Score Table

Problem	Points	Out of
1.		10
2.		10
3.		10
4.		10
5.		12
6.		12
7.		12
8.		12
9.		12
Total:		100

1. (10 marks) Solve

$$y' = e^{x+y}, \ y(0) = 0$$

and find the maximum interval on which the solution is valid.

2. (10 marks) Using the substitution $v = y^{1/2}$, find the solution $y(x)$ of

$$x^2 y' + 2y = e^{1/x} y^{1/2}, \quad y(1) = 4.$$

3. (10 marks) Solve implicitly,

$$\frac{dy}{dx} = \frac{y}{x + x^6}, \quad y(1) = 1.$$

(HINT: do not use separation of variables!)

4. (10 marks) Find the general solution $y(x)$ of

$$\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 8\frac{d^2y}{dx^2} = 2x.$$

5. (12 marks) Find the solution $y(x)$ of

$$y'' - 2y' + 2y = \frac{e^x}{\cos x}, \quad y(0) = 2,\ y'(0) = 1.$$

6. (12 marks) Use Laplace transforms, and the table which follows, to solve

$$y'' + 2y' + 2y = 2t + 3\delta_4(t), \;\; y(0) = 1, y'(0) = -2.$$

function $f(t)$	Laplace transform $F(s)$
1	$1/s \;\; (s > 0)$
t^n	$n!/s^{n+1} \;\; (s > 0)$
e^{at}	$1/(s-a) \;\; (s > a)$
$\sin at$	$a/(s^2+a^2) \;\; (s > 0)$
$\cos at$	$s/(s^2+a^2) \;\; (s > 0)$
$e^{-at}f(t)$	$F(s+a)$
$u_a(t) = u(t-a) \; (a \geq 0)$	$e^{-as}/s \;\; (s > 0)$
$\delta_a(t) = \delta(t-a) \; (a > 0)$	e^{-as}
$u(t-a)f(t-a)$ or $u_a(t)f(t-a)$	$e^{-as}F(s)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) \ldots - f^{(n-1)}(0)$
$f * g(t) = \int_0^t f(r)g(t-r)\,dr$	$F(s)G(s)$

7. (12 marks in total) You are given the following matrix A with its row reduced echelon form R:

$$A = \begin{pmatrix} 2 & 0 & -1 & 1 & 4 \\ 2 & 0 & 3 & 5 & 12 \\ 1 & 0 & 1 & 2 & 5 \end{pmatrix}, \qquad R = \begin{pmatrix} 1 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) (4 marks) Write down a basis for the row space of A and express all rows of A as linear combinations of the basis vectors.
(b) (4 marks) Find a basis for the column space of A and express the fifth column of A in terms of that basis.
(c) (4 marks) Find an orthogonal basis for the column space of A.

8. (12 marks in total) Consider the symmetric matrix $A = \begin{pmatrix} -1 & 4 \\ 4 & 5 \end{pmatrix}$.

(a) (4 marks) Find the eigenvalues and the corresponding eigenspaces.

(b) (3 marks) Find an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

(c) (3 marks) Find the matrices of orthogonal projection onto each of the two eigenspaces of A.

(d) (2 marks) What is the relationship between A and the two projection matrices you have found in (c). Verify that your answer is correct.

9. (12 marks in total) Consider $A = \begin{pmatrix} 2 & -5 \\ 2 & -4 \end{pmatrix}$. The vector $\begin{pmatrix} 1+2i \\ 1+i \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue $-1+i$.

(a) (4 marks) Write down a basis, in vector form, of real solutions of the system

$$\begin{aligned} x_1' &= 2x_1 - 5x_2 \\ x_2' &= 2x_1 - 4x_2. \end{aligned}$$

(b) (3 marks) Find the functions $x_1(t)$ and $x_2(t)$ which satisfy the above system as well as the initial conditions $x_1(0) = 1,\ x_2(0) = 0$.

(c) (3 marks) Write down an expression for e^{At} involving the product of specified real matrices and their inverses.

(d) (2 marks) How do solutions of the system in (a) behave as $t \longrightarrow \infty$?

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